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## Faculty Working Papers

PRICING OF LIQUIDITY FOR PREFERRED STOCKS ON  
THE NEW YORK STOCK EXCHANGE

Frank K. Reilly, Professor, Department of Finance

#662

**College of Commerce and Business Administration**  
**University of Illinois at Urbana-Champaign**

- Eiglier, Pierre and Eric Langeard (1977), "A New Approach to Service Marketing," in Marketing Consumer Services: New Insights, Pierre Eiglier, Eric Langeard, Christopher H. Lovelock, John E. G. Bateson, and Robert F. Young. Cambridge, MA: Marketing Science Institute, Report #77-115 (November), 31-58.
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## Faculty Working Papers

THE PRESENT VALUE OF POPULATION GROWTH IN THE  
WESTERN WORLD

Julian L. Simon, Professor, Departments of  
Business Administration and Economics

#666

College of Commerce and Business Administration  
University of Illinois at Urbana-Champaign





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Summary

Population policy discussions have been influenced by conventional steady-state growth theory in which technical progress is exogenous, implying that faster population growth causes a lower rate of consumption. Making technical progress endogenous in the manner of Verdoorn, Kaldor, Arrow, or Phelps, however, is shown to lead to the opposite implication, that higher population growth causes a higher rate of growth of consumption, though this inference has not been drawn by the authors of these models.

Steady-state equilibrium analysis is not appropriate for policy decisions, though, because when a nation chooses one or another population growth rate, it begins with the same endowment of capital and people and techniques no matter what the population growth rate chosen. Therefore the appropriate analysis is one which compares the result of two or more growth rates beginning from that initial position.

The paper analyzes the supply and demand of knowledge, and on those considerations derives the most plausible technical progress functions, which turn out to be mostly variations on a function of Phelps. The effects of various rates of population growth are then simulated with these variations specification and parameters.

The chief result is that with virtually every variant, faster population growth shows better consumption results with discount rates up to 5-10%, a level which is far above the long-run adjusted riskless rate; at higher discount rates lower (or negative) population growth rates have higher present values. If pensions were brought into the analysis, a la Modigliani, Tobin, and Samuelson, higher population growth rates would seem even more beneficial at low discount rates, and would have an advantage over lower population growth rates at discount rates even higher than 6-10%. And even at very high discount rates, lower population growth rates imply present values only a bit above those for higher population growth rates. The advantage is overwhelmingly with higher population growth in this growth-theoretic analysis.



# THE PRESENT VALUE OF POPULATION GROWTH IN THE WESTERN WORLD

Julian L. Simon\*

## I. INTRODUCTION

Whether additional people are good or bad, from an economic point of view, depends entirely upon whether people's welfare in only the near future is taken into account, or whether the long-run future also is allowed to weigh heavily in the judgment.

It is an advantage of economics over the other social sciences that with the concept of discounting one can compare offsetting events that will occur at separate moments, and arrive at a single-valued judgment about the net value in the present of that stream of future events. But this powerful tool has not hitherto been put to work in the evaluation of long-run population growth, a situation where evaluation over time is particularly crucial.\*\* To put this tool to work, and thereby to arrive at some overall judgments about the social value of population growth within models that appropriately embody the most important impact of population growth and size--the effect of a larger population and a larger market upon technical change--is the aim of this paper. Analysis of the nature of previous growth models, and of models newly proposed here, is a conjoint part of the work.

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\*I benefited from several discussions of the topic with Mark Browning, and I enjoyed an early talk about this general field with Leonard Mirman.

\*\*Enke's (e.g., 1966) focus is quite different than that of this paper, being on the evaluation of the present worth of a single person or cohort, rather than the long run of growth theory. (Also, Enke's work is internally inconsistent; see Simon, 1969; 1977, Chapter 20).

Enke's focus is quite different than that of this paper, being on the evaluation of the present worth of a single person or cohort, rather than the long run of growth theory. (Also, Enke's work is internally inconsistent; see Simon, 1969; 1977, Chapter 20).

The main story may be summarized briefly as follows: (1) At a very high discount rate--that is, where only the present and near future matter in the reckoning--any additional people have negative value, because there are some short-run negative externalities for the "incumbents" due to (a) transfer payments for social services, and (b) reduction in per worker output due to capital dilution. The latter is the entire corpus of Malthus' reasoning, diminishing returns to fixed capital. The differences are small among population growth rates with respect to both (a) and (b), however. (2) Conventional growth theory dynamizes the Malthusian proposition, but the conclusion is straightforwardly the same: a higher population growth rate implies lower consumption per person under all circumstances, because more of output must go into investment to maintain an equilibrium growth rate; a negative population growth rate is best of all, all the way to zero births. (3) Any growth model that makes the amount of technical change a function of absolute population size, market size, or capital stock will eventually have higher consumption with faster rather than slower population growth, because of the cumulative nature of knowledge;\* this is also true of most models in which the rate of technical progress is made a function of the rate of change of capital or output (but these models are less germane theoretically). The date at which consumption becomes higher with faster population growth can be closer or farther in the future depending on the model and the parameters. (4) Simulations with a variety of parameters of the various absolute-size models show that faster population growth almost always has a higher

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\*Some knowledge grows less useful as it gets older. But it is most unlikely that knowledge obsolesces enough to matter in this context.

present value of future consumption at discount rates of below 6-10%; at higher discount rates, lower (or negative) population growth generally has a higher present value. This suggests a much higher return to additional people than the long-run private riskless market rate of 2-3%.

The center of any analysis of the effect of additional people on income through the production of knowledge is this indigestible kernel: No matter how small the contribution of knowledge of the additional person, it will someday--though perhaps a long time in the future--inevitably lead to income per capita being higher than otherwise, *ceteris paribus*.

There is also a subplot to the story: In a modern society, workers transfer amounts of income to retired persons that are large in comparison to the transfers to children. This means that for any given person it is beneficial if some other person has more children, *ceteris paribus*; this effect raises the discount rate at which the present value of population growth is positive above what it otherwise would be with any given model. This paper concentrates on the main story line, however. Disregarded in the calculations are transfers to retired persons and transfers for child services, as well as negative environmental adjustment costs, positive environmental externalities of all sorts (especially those that decrease the entropy of the earth), and changes in work and savings patterns due to larger numbers of dependents.

The paper focuses on the more-developed world (MDC's) as a whole (the West plus Japan and Oceania), in which technological change is created as well as adopted from elsewhere: with the passage of time, the poorer part of the world will also contribute heavily to the rest of the world's technical level, however, as well as simply being mostly a recipient as at

present. Consideration of the MDC's as a whole also sidesteps a possible coat-tails strategy for any individual country.

## II. THE THEORY OF SUPPLY, DEMAND, AND THE APPROPRIATE TECHNICAL CHANGE FUNCTION

Though this section will end still a long way from a satisfactory theory, or theory-based model, of technical change as part of the growth process, it is worthwhile to at least put the matter in a general theoretical context.

Though economists generally begin analyses with the forces of supply and demand, it has not been so with technical change in the context of growth models. Previous endogenous technical-change models have worked only with investment as a variable. And though capital investment that embodies technical change may be seen as a carrier of new techniques, it is identified with neither supply nor demand. (Of course cumulative capital may also be viewed as an empirical proxy for total output a la Arrow, 1962.)

### Notation

S = supply of new knowledge and technical advance

L = population (= labor force in this context)

s = saving rate, as a proportion of output

n = rate of growth of labor force

Y = total output

K = stock of capital

$v = \frac{Y}{K}$  = reciprocal of capital-output ratio

w = the wage rate

r = rate of return on capital

$M$  = the level of labor-augmented technology, which may be thought of as productivity per worker

$E$  = disequilibrium gap in capital-output ratio

$z$  = adjustment factor in savings function

$a, b, c, e, \alpha, \beta, \gamma, \Delta, \phi, \psi, = \text{constant}$

### The Supply Side

The supply of technical changes is relatively easy to analyse. Supply clearly depends on the number of persons, trained or untrained technically, available to produce or adapt new ideas; this is the labor force,  $L$ .\*

The supply of technical change also depends upon the level of education and training of the labor force, both the amount of specific training (which could be indexed by the supply of scientists and engineers) and the unspecific general education (which can be indexed by the mean education of the society) that leads to such important innovations as new organizations. Because of the close relationship between per capita

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\*This proposition--that, *ceteris paribus*, more people mean more inventions, technical change, and productivity increase--seems as self-evident to me as any economic proposition can be. It is also so obvious to Machlup that he handles it in twelve words: "the supply of labor--the chief input for the production of inventions." (1962, p. 143). Kuznets, and Petty long before him, asserted the same idea without thought of contradiction.

income and mean education, the general level of education may also be indexed by per capita income,  $\frac{Y}{L}$ .

Though scientific knowledge is not, by any means, the same as technical change, it is a key precursor of it—both "spontaneous" and "induced" technical change. It is therefore relevant and important that in a cross-section of countries, population size and per capita income together are an excellent explanation of the amount of scientific activity (Price 1967, 1971, 1975; Love and Pashute, 1978).

The production of new knowledge requires the existence of a stock of knowledge. Hence the quantity of existing knowledge, both in storage in libraries and in action in the level of technical practice in the economy, constrains the amount of new knowledge that can be created at any moment by various numbers of persons. This implies diminishing returns to the fixed stock of knowledge at any moment.\* (This effect is not seen in a cross-national analysis because all countries have roughly the same access to the existing body of knowledge.) Therefore an appropriate technical change function should contain an argument for the existing stock of knowledge.

The total number and variety of physical stimuli--objects and processes--that exist in a society at a given moment must also influence the supply of inventions, as the weather influences crops. Per capita income

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\*Kuznets (1960, p. 328) does not accept that returns must diminish even in a static sense, because "creative effort flourishes in a dense intellectual atmosphere, and it is hardly an accident that the loccus of intellectual progress (including that of the arts) has been preponderantly in the larger cities." Higgs (1971) and Kelley (1972) find some statistical verification for this point.



is one measure of this factor, but total income  $Y$  is likely to be another. This jibes with Verdoorn's Law, which makes the change in productivity a function of change in income, and with Clark's data (1967, on p. 265) which implicitly makes the level of productivity a function of total income. It also fits the learning-by-doing data.\*

Within a given economy, the supply of inventors and inventions clearly is a function of the returns in profits to the firm and payments to investors; some poets and mathematicians will temporarily turn their fine minds to productivity increases (or to pornographic novels) if the prize offered is large enough.\*\* And more prosaically, some persons now working at non-R&D jobs—say, engineers out on the road selling existing products, and technicians working on existing electronic equipment—will move to R&D jobs and departments as the wage rises in R&D. This effect is mostly not relevant to a study of the effect of different rates of population growth upon the rate of invention. A related idea will be explored, however, in the section on the demand side.

The supply function may therefore be written

$$(1) \quad S = f(L, Y, \frac{Y}{L}, M)$$

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\*One may wonder how the studies showing that the rate of learning declines with output (e.g., Barkai and Levhari, 1973; Levhari and Sheshinski, 1973, Baloff, 1966, and references cited therein) fit in here. The rate of learning in a given product situation may decrease, but there may still be changes in the processes which restart a high-rate learning process, and increase the overall rate of learning which is consistent with the envelope curve in Figure 1.

\*\*Machlup (1962) makes this point well.

If one assumes that the mean level of education increases the average individual's knowledge-producing capacity, though not linearly, and if we notice that population size multiplied by per capita income equals total output, we can write a more explicit supply function

$$(2) \quad S = f(Y^\gamma M^\Delta) \text{ with } \gamma < 1 \text{ and } \Delta < 1.$$

It is important to notice that there need not be diminishing returns over time to additional people, because the stock of knowledge with which people may combine their creative talents grows with time. And I believe the argument is strong that returns will increase. Kuznets makes an argument for increasing returns on two grounds: (1) the stimulative effect of dense environment mentioned earlier, and (2) "interdependence of knowledge of the various parts of the world in which we human beings operate" (p. 328); e.g., discoveries in physics stimulate discoveries in biology, and vice versa. Kuznets discounts the possibility of diminishing returns because "the universe is far too vast relative to the size of our planet and what we know about it" (p. 329). Machlup suggests that "every new invention furnishes a new idea for potential combination with vast numbers of existing ideas... [and] the number of possible combinations increases geometrically with the number of elements at hand" (1962, p. 156). It is this latter idea of an increasing number of possible permutations of the available elements of knowledge as the stock increases, when combined with the idea of a reduced likelihood of duplicate discoveries as the number of possibilities

increases faster than the number of potential knowledge producers, that seems most compelling to me.\*

### The Demand Side

The demand for improvements in productivity that will cut costs clearly depends upon the scale of the industry in which the improvements can be put to work. We observe more total R&D in large industries than in small ones, and it is reasonable to expect that a larger country will have more total R&D than a small one, in part because of the greater scope of utilization of improvements; this is confirmed by the Price and Love-Pashute data mentioned earlier.

It may be illuminating to consider an example of how demand for R&D rises as overall demand is expected to rise from additional persons. Imagine that your firm considers producing a new sort of reading-talking computer for the blind. Right now your financial projections are just below the break-even point. If you are suddenly informed that the population of potential users in 2, 4, 8 and 16 years from now will be 50% larger than you had entered into your calculations, your present-value computation will now be positive (on any reasonable assumptions about potential competitive behavior). You will now have a greater demand for R&D workers.

With respect to investment and productive equipment, the effective demand for new improvements is an accellerator function; if output is

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\*I am presently building a micro model of the inventive process based on these processes, and I hope to be able to make more conclusive statements about this matter.

stable and can be supplied by existing plant and equipment, there will be little demand for the improvements that can only come with new investment. (But the Horndal effect makes clear that productivity change occurs even without the installation of new equipment; see Arrow, 1962, and David, 1975).

Taken together, then, the demand for economic improvement may be seen as a function of total output, change in output, and per capita income (or per capita output)

$$(3) \quad D = f(Y, \dot{Y}, \frac{Y}{L}).$$

There is another facet of the demand side that is harder to fit with conventional economic thinking, because it requires the concepts of needs and aspirations. The concept of need is a subjective rather than objective idea, but felt need certainly can stimulate innovative activity. "Necessity is the mother of invention"—few would be so rigid in their adherence to traditional economic categories as to deny all meaning to this notation. Implicit in this idea is that at a given moment there is a gap between the individual's rate of output, or stock of inputs, such that the individual has a greater-than-otherwise motivation to find a new idea to increase the effective stock of inputs, or the manner in which the inputs can be combined, so as to increase the rate of output. Finding and developing such a new idea may require additional labor time on the part of the individual, and/or additional "effort," whatever the meaning of the latter term.

Surely this is the process that occurs in wartime or in economic downturn or in other emergency, as a host of anecdotes show. I know

of no statistical evidence linking innovation and level of "need." But there is ample evidence, both across nations at present, and within nations over time, that lower actual average income or wages leads to more hours worked per week. There is also an observed relationship between number of children and number of work hours. (See Simon, 1977, Chap. 4, and Lindert, 1978, App. B). These phenomena can be understood with the tradeoff between income and leisure described for us by Hicks (1932) and in more detail for the peasant farmer by Chayanov (1925/1966). It seems reasonable that there should be a similar tradeoff between income aspirations, and leisure (in the sense of a respite from the sort of effort required to create and develop new innovations). The smaller the homemaker's budget, the more ingenious one would expect her or him to be in inventing ways to pad the hamburger with cheaper filler; and the smaller the home, the more ideas for utilizing the space efficiently that you would expect from the homemaker. (The ingenuity of sailors in utilizing every nook and cranny on submarines and other small navy ships is a marvel).

It is also reasonable to extend this line of thought from need and actual income to income aspirations--an effect, however, that runs in the opposite direction from actual income. The higher one's actual past income, the higher is one's wealth and the lower is one's objective need for present income. But the higher one's actual past income, the higher one's aspirations for present income. It would be possible to study the effect of aspirations econometrically by using the actual income in a given year, or the income that the trend of past income would lead one to expect, as an independent proxy variable.

### III. WHAT IS THE APPROPRIATE GROWTH MODEL FOR THE UNDERSTANDING OF POPULATION GROWTH?

#### Critical Assessment of Existing Models

A brief review is necessary to set the scene. Malthus is the fount, and in his theoretical system an additional person has a straightforward negative effect on the income of others due to capital dilution and diminishing returns: more workers on the same acres mean less output per worker.

In the simplest Harrod-Domar growth model there is no technological change at all. And the result is much the same as that of Malthus, and for the same reason: with more people, more investment is necessary to maintain the same levels of capital and of output per person, which implies lower consumption per person. Growth theory that introduces constant exogenous technological change leads directly to the same conclusion: faster population growth implies lower consumption.

The main corpus of growth theory with respect to population growth can fairly be summarized as a no-complications dynamization of Malthus' capital dilution with a simple conclusion: more people imply lower income.\* You may check the accuracy of this survey in works on population growth by Phelps (1972) or Pitchford (1974), as well as in such general studies of growth as Solow (1970), Brems (1973) and Dixit (1976).

Furthermore, it is amply clear that this main stream of growth theory has had influence on the policies of nations and on public opinion; the former is well-documented in Piotrow (1973).

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\*The following paragraphs draw upon a companion paper (Simon, 1979).

But a powerful empirical fact enters here: Models implying that population growth is negatively associated with economic growth are falsified by the data for the past century and half century for those western-style countries for whom Kuznets could find data, as well as the cross-sectional data on economic growth since World War II (for a summary see Simon, Chapter 4). This fact means that the models without endogenous technical change are without value for the understanding of population growth's effects.

It is reasonably easy to show that in a comparison of two populations with different rates of labor-force growth (with optimum savings ratios) that are already on the equilibrium growth path, faster labor-force growth implies higher consumption--as long as technical progress is a function of the size of the labor force on total output, even to the slightest degree. Consider a situation in which technical progress does not depend upon either labor force or total output; if so, technical progress will be the same, and the rate of growth of per-capita output will also be the same, for every rate of population (labor force) growth, though consumption will be lower with higher population growth due to the higher warranted savings rate. But if technical progress is faster with a higher population growth rate—as it is with function (6a) because faster population growth implies faster increase in aggregate income and hence in capita--then the rate of growth of per capita output must be faster with higher population growth. And this must therefore eventually (no matter how slight the dependence of technical progress on population

or output) lead to higher levels of per capita output and consumption.

This is the argument in a nutshell.\*

Before we begin to examine possible technical progress functions, we must consider which facts--stylized or otherwise--we wish the function to fit. The most important fact is that technical progress has been at least proportional to population size throughout human history; when population was very small 10,000 or 20,000 years ago, the amount and the rate of technical progress was small compared to later periods when population (as well as the stock of knowledge) was larger. And there is no persuasive reason to expect this to change in the future. There are at least some writers on the physical sciences (see Rescher, 1978 and citations therein) who argue that a given quantity of resources brings decreasing amounts of knowledge as time progresses. Whether or not this is true for the physical sciences--and there are vast and perhaps insuperable problems in comparing the value of different discoveries as well as strong philosophical arguments against as well as for this position--this particular trend simply does not find analogy in the rate of technical progress as measured by various proxies for national income. And even if there is a limit to the number of "laws" that can be discovered in the physical sciences, there is certainly no such limit in the social and economic sciences, because increasing institutional complexity

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\*The comparison of golden-age growth paths is not the relevant comparison for a given society's policy choice at a given moment, however. Rather, the society wishes to evaluate its future streams of costs and benefits with different rates of population growth, given its present endowment of capital and level of income. That comparison is the subject of another paper.



creates new material for study and understanding. Therefore it seems reasonable that the rate of change of technical progress should be at least proportional to the size of population, the stock of knowledge, the size of GNP, the stock of capital, and so on.

There remains the question whether the rate of technological progress should be greater than the rate of growth of population. Solow (1957) calculated that the yearly rate of change of the technological coefficient in his model went from 1% to 2% over the 20 years from the (median of the) first half of his study period to the (median of the) second half of his period (1909-1949). Fellner computed rates of growth of productivity of 1.8% for 1900-1929; 2.3% for 1919-1948; and 2.8% for 1948-1966 (1970, pp. 11-12). And the Kendrick-NBER data show that the rate of growth of output per unit of labor input rose .012% per year from 1890 to 1957, which means a rise from 1.62% yearly at the starting point to 2.38% yearly at the endpoint.\*

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\*One might object that the period covered by these U.S. data is only a small segment of history. But this segment is all we have that is quantitatively measured--and it is the same segment used when writers on growth cite data on capital-output ratios. As to a drop in the most recent years, it is very seldom sensible to read changes in long-run secular trends from a few years' data; doing so has produced more wrong predictions than any other single bad practice in social science (e.g., the predictions of shortages and scarcities starting in 1973). Furthermore, Denison's close examination of the recent data persuade him that "lack of advances in knowledge was not responsible for most of the drop" in the growth rate of national income per person employed since 1974 (1978, p. 12). The entrance of less-skilled and less-experienced persons during a period of rapid growth of the kind here has been important, as have a shift to services and the need to invest in environmental and health protective advances which do not appear in GNP and productivity figures. Considered by itself, manufacturing has been making "a good advance" (Federal Reserve Bank, 1979, p. 7).

Some growth theorists however have made the amount of technological change endogenous.\* Kaldor (1957, and later a bit differently with Mirrlees, 1962) was impressed by "Verdoorn's Law" relating technical change to change in output, and he modeled changes in productivity as a function of investment:

$$(4) \quad \frac{M_t - M_{t-1}}{M_{t-1}} = f \quad \frac{K_t - K_{t-1}}{K_{t-1}}$$

But this function - like all other functions that work with changes in levels, rather than with the absolute quantities themselves - has zero technical change when the labor force is constant, which is falsified by the Horndal data and all everyday observation.

Alchian took note of the fact that production of such products as airframes improves in the course of production. He then introduced this "learning-by-doing" insight into economic theory, distinguishing among various sorts of economies of scale (1949/1963; 1959). Arrow (1962) then built an explicit technical-progress function upon this foundation, and--unlike Kaldor--went over to cumulative investment as the carrier (or the proxy for the carrier) of embodied technical progress:

$$(5) \quad M_t = cK_t^\phi$$

where  $\phi$  is a constant comparable to the coefficient of serial numbers in learning-by-doing studies, and is of the order of .2. But Arrow's function is not fundamentally different than is Kaldor's, as we see by writing from (5)

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\*I exclude from consideration the body of literature concerning the direction of technical change as an endogenous matter, e.g. von Weiszacker (1966), and Nordhaus (1969).

$$(5a) \quad \frac{M_t - M_{t-1}}{M_{t-1}} = \frac{cK_t^\phi - cK_{t-1}^\phi}{cK_{t-1}^\phi}$$

and comparing to equation 4.\* Furthermore, Arrow's function is really not a good transcription of the learning-by-doing idea; a more realistic rendering will be discussed below.

Phelps (1966) suggested a most interesting model which (approximately) reduces to

$$(6) \quad \frac{M_t - M_{t-1}}{M_{t-1}} = h\left(\frac{L_{t-1}}{M_{t-1}}\right)$$

where  $h$  is a concave function. The functions explored in the next section are similar to Phelps' function, though developed from different arguments. Hence I shall not discuss Phelps' function further at this point.

In previous work (1977, Chapter 6) I made the rate of change of technology a function of, alternatively, the labor force, and total output

$$(7) \quad \frac{M_t - M_{t-1}}{M_{t-1}} = aY_{t-1}^\phi \quad \phi < 1$$

and

$$(8) \quad \frac{M_t - M_{t-1}}{M_{t-1}} = bL^\psi \quad \psi < 1.$$

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\*Eltis (1973) and Shell (1966) have discussed other variants of Kaldor's function.

I do not any longer feel that there is evidence supporting a function that is so sensitive as to yield an increasing rate of growth of knowledge in response to a steady rate of growth of population.

The difference of degree between my function (8) and Phelps' formulation (6) is a matter of the size of the impact of existing M. This may be seen by writing both functions in Cobb-Douglas production function format

$$(6a) \quad M_t - M_{t-1} = M_{t-1}^{.5} L_{t-1}^{.5},$$

where the exponents come from the rest of Phelps' discussion, and

$$(8a) \quad M_t - M_{t-1} = M_{t-1} L_{t-1}^{\phi}, \quad .1 \leq \phi \leq .5.$$

Phelps' function, homogeneous of degree one, yields steady-state growth (including a constant rate of change of M) whereas my (8a) yields an increasing rate of change of M.

From here on the dependent variable on the left hand side will be written as the absolute quantity of change in the technical level, rather than the rate of change, an alteration I consider important. The latter is a more basic and more illuminating way to view the matter. This can be seen merely as a matter of algebra, i.e., re-arrangement, but the

reader's interpretation may well be affected by the way the algebra is written.\*\*

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\*\*Leontief argues strongly that different algebraic modes of expression of a proposition have importantly different psychological effects on the scientific reader:

[I]n the actual process of scientific investigation, which consists in its larger part of more or less successful attempts to overcome our own intellectual inertia, the problem of proper arrangement of formal analytical tools acquires fundamental importance.

... The degree of mental resistance which accompanies the use of one or another formal pattern is furthermore rather closely (although also only "statistically") and positively correlated with the chance of committing logical mistakes. Mistakes of this kind may manifest themselves either in the inability to perceive the "evidence" of a correct argument or in the practically much more dangerous readiness to be convinced by a false one. (Leontief, 1966, pp. 59-60)

#### IV. A MORE COMPLETE AND APPROPRIATE MODEL

Section II discussed the general theory of the relationship of population to the technical change function. Now after the critical review in Section III we are in position to start from scratch and discuss in greater detail which model should be considered appropriate for simulating the effect of various rates of population growth.

The supply-side theory suggests rather straightforwardly that  $L$  and  $M$  are key arguments. The demand-side theory suggests  $Y$  as the fundamental factor, along with  $\frac{Y}{L}$ ,  $\Delta Y$ , and  $[(\frac{Y}{L})_t - (\frac{Y}{L})_{t-1}]$ . For simplicity, however, we shall not include the latter two arguments in the present work, though later work should study whether their inclusion affects the results.

It is reasonable to assume that the arguments interact in a multiplicative fashion, with each of the factors subject to diminishing returns. Expressing for clarity's sake the left-hand-side as the absolute amount of advance (rather than the rate of change) in a given period, the general function is then

$$(10) \quad M_t - M_{t-1} = a L_t^\gamma M_{t-1}^\Delta (\frac{Y}{L})^\psi Y^\phi.$$

Runs were made with a variety of parameters, adding alternatively to less than unity, unity, and more than unity;  $L$  and  $M$  are given much larger exponents than  $(\frac{Y}{L})$  and  $Y$  in most cases. I also ran a variety of functions without one, two, or three of the arguments in equation 7 (noting that  $\frac{Y}{L}$  can be reduced to  $Y$  and  $L$  when both the latter are also in the equation).

The rest of the simulation is as simple as possible: A Cobb-Douglas production function with technological progress introduced (for clearest understanding) as a multiplier of labor

$$(11) \quad Y_t = \frac{1}{v} K_t^\alpha (M_t L_t)^\beta, \text{ and}$$

the savings function either

$$(12) \quad K_t = K_{t-1} + sY$$

or the following set of equations

$$(13) \quad K_t = K_{t-1} + sY + zY$$

$$(14) \quad E = \beta \left( \frac{Y_{t-1}}{K_{t-1}} \right) - \beta \left( \frac{Y_{t=0}}{K_{t=0}} \right)$$

$$(15) \quad z = [\text{sign of } E] [1 - e^{-4.2|E|}]$$

which makes saving a function of the difference between the original output/capital ratio and the current output/capital ratio.

I have then simulated the effects of various parameters and various rates of growth  $\underline{d}$  of the labor force

$$(16) \quad L_t = L_{t-1} + dL_{t-1}, \quad d = -1\%, 0\%, 1\%, 2\%$$

The following combinations of savings rates and rates of growth of the labor force provide most of the desired comparisons:  $\dot{L} = 1\%$ ,  $s = 2\%$ ;  $\dot{L} = 1\%$ ,  $s = 4\%$ ;  $L = 2\%$ ,  $s = 4\%$ .

## RESULTS

### Procedure

The results of various simulation runs are shown in Table 2. The results are present-value calculations of the stream of consumption at different discount rates. An example of the underlying data,

year by year for ten years, and then at ten and fifty year intervals is shown in Table 1.

- - - - -  
Tables 1 and 2  
- - - - -

Present values were calculated at the end of 300 years, at which time the present value has virtually ceased to grow with the addition of more periods, except at a zero discount rate. For those runs in which the computer ceased functioning short of 300 years due to some magnitude in the simulation becoming too large (usually M, though sometimes C), present values were computed for all L at the highest of 250, or 200, or 150 years for which the calculations are complete. For some runs in which it was interesting to look for convergences and golden-age paths, the model was run for 600 years (data not shown here).

### Findings

The general picture for any particular model is that the yearly consumption level starts out lower with a higher population growth rate than with a lower population growth rate, but becomes higher somewhere around the twenty-fifth year after the entry of the additional persons into the labor force, on the average. This is the same picture that emerged in previous works (Simon, 1977, Chap. 6).

For purposes of decision about population policy, however, we must know the trade-off between present and future consumption, just as when making decisions about dams, environments, monuments, and other matters that have very-long-run ramifications. An appropriate manner to think about this problem is to examine the present values of the alternatives at a discount rate deemed reasonable, and select that alternative that



$$M_t - M_{t-1} = a L_{t-1} M_{t-1}^{.5}$$

TABLE 1

YEAR	K	Y	$\frac{Y}{L}$	$\frac{K}{Y}$	M	$\frac{C}{L}$
0	.100E+04	.200E-01	.500E+00	.200E+01	.100E+01	.500E+00
1	.102E+04	.205E-01	.514E+00	.199E+01	.101E+01	.493E+00
2	.104E+04	.207E-01	.527E+00	.198E+01	.102E+01	.487E+00
3	.106E+04	.208E-01	.542E+00	.196E+01	.103E+01	.490E+00
4	.108E+04	.209E-01	.556E+00	.195E+01	.104E+01	.494E+00
5	.110E+04	.211E-01	.572E+00	.194E+01	.105E+01	.497E+00
6	.113E+04	.212E-01	.587E+00	.193E+01	.106E+01	.501E+00
7	.115E+04	.213E-01	.603E+00	.192E+01	.107E+01	.504E+00
8	.117E+04	.215E-01	.620E+00	.190E+01	.108E+01	.508E+00
9	.120E+04	.216E-01	.637E+00	.189E+01	.110E+01	.512E+00
10	.122E+04	.217E-01	.655E+00	.188E+01	.111E+01	.516E+00
20	.149E+04	.230E-01	.864E+00	.178E+01	.123E+01	.558E+00
30	.181E+04	.242E-01	.115E+04	.169E+01	.138E+01	.609E+00
40	.221E+04	.254E-01	.154E+04	.162E+01	.155E+01	.668E+00
50	.269E+04	.265E-01	.207E+04	.155E+01	.175E+01	.739E+00
60	.320E+04	.275E-01	.281E+04	.149E+01	.198E+01	.822E+00
70	.400E+04	.285E-01	.384E+04	.144E+01	.225E+01	.921E+00
80	.488E+04	.294E-01	.527E+04	.140E+01	.258E+01	.104E+01
90	.594E+04	.303E-01	.727E+04	.136E+01	.296E+01	.117E+01
100	.724E+04	.311E-01	.101E+05	.133E+01	.341E+01	.134E+01
110	.883E+04	.318E-01	.141E+05	.130E+01	.395E+01	.153E+01
120	.108E+05	.325E-01	.197E+05	.127E+01	.459E+01	.176E+01
130	.131E+05	.332E-01	.278E+05	.125E+01	.535E+01	.203E+01
140	.160E+05	.338E-01	.393E+05	.122E+01	.625E+01	.236E+01
150	.195E+05	.343E-01	.558E+05	.121E+01	.735E+01	.275E+01
200	.525E+05	.365E-01	.338E+06	.114E+01	.170E+02	.619E+01
250	.141E+06	.379E-01	.218E+07	.110E+01	.416E+02	.148E+02
300	.380E+06	.388E-01	.147E+08	.107E+01	.105E+03	.371E+02
350	.102E+07	.394E-01	.102E+09	.105E+01	.272E+03	.954E+02
400	.275E+07	.398E-01	.716E+09	.104E+01	.716E+03	.249E+03
450	.741E+07	.400E-01	.510E+10	.104E+01	.190E+04	.650E+03
500	.200E+08	.402E-01	.365E+11	.103E+04	.506E+04	.174E+04
550	.537E+08	.403E-01	.263E+12	.103E+04	.135E+05	.470E+04
600	.145E+09	.403E-01	.190E+13	.131E+05	.363E+05	.126E+05

TABLE 2

 $PV_{t=300}$  for the Equation

$$M_t - M_{t-1} = aL_t^Y M_{t-1}^{\Delta} \left(\frac{Y}{L}\right)_{t-1}^{\psi} Y_{t-1}^{\phi}$$

Row #	Exponents for				Discount Rate							
	L	M	$\frac{Y}{L}$	Y	L	S	0%	2%	4%	6%	8%	10%
1.1	.5	.5	NA	NA	1%	2%	696	39.2	14.9	9.20	6.67	5.24
1.2	"	"	"	"	2%	4%	2,110	52.3	15.4	9.20	6.61	5.17
1.3	"	"	"	"	1%	4%	917	45.1	15.8	9.52	6.81	5.30
2.1	.3	.3	NA	NA	1%	2%	451	35.6	14.6	9.13	6.65	5.23
2.2	"	"	"	"	2%	4%	666	38.8	14.7	9.07	6.57	5.15
2.3	"	"	"	"	1%	4%	585	40.5	15.5	9.44	6.78	5.29
3.1	.7	.7	NA	NA	1%	2%	1,740	48.5	15.3	9.29	6.70	5.25
3.2	"	"	"	"	2%	4%	4,800	261	17.7	9.41	6.67	5.20
3.3	"	"	"	"	1%	4%	+E1 2,350	57.5	16.4	9.62	6.84	5.31
4.1	.9	.9	NA	NA	1%	2%	3,700	193	16.7	9.42	6.74	5.27
4.2	"	"	"	"	2%	4%	+E1 6,580	1,740	4,180	9.26	8.65	5.23
4.3	"	"	"	"	1%	4%	+E7 5,190	+E5 256	+E3 18.2	9.77	6.88	5.33
5.1	.5	.9	NA	NA	1%	2%	2,360	51.8	15.4	9.30	6.70	5.25
5.2	"	"	"	"	2%	4%	1,300	514.	18.2	9.36	6.66	5.19
5.3	"	"	"	"	1%	4%	+E5 3,190	62.0	16.5	9.63	6.84	5.31
6.1	.4	.6	NA	NA	1%	2%	696	39.2	14.9	9.20	6.67	5.24
6.2	"	"	"	"	2%	4%	1,790	49.6	15.3	9.18	6.60	5.17
6.3	"	"	"	"	1%	4%	917	45.1	15.8	9.52	6.81	5.30
7.1	.6	.4	NA	NA	1%	2%	696	39.2	14.9	9.20	6.67	5.24
7.2	"	"	"	"	2%	4%	2,420	55.0	15.5	9.22	6.62	5.18
7.3	"	"	"	"	1%	4%	917	45.1	15.8	9.52	6.81	5.30
8.1	.5	1.0	NA	NA	1%	2%	6,170	69.6	15.6	9.33	6.71	5.26
8.2	"	"	"	"	2%	4%	5,500	1,470	375	10.2	6.67	5.19
8.3	"	"	"	"	1%	4%	8,430	86.4	16.8	9.66	6.85	5.32
9.1	.5	1.1	NA	NA	1%	2%	1,140	397	16.9	9.36	6.72	5.26
9.2	"	"	"	"	2%	4%	1,080	6,980	4,070	2,130	9,950	466
9.3	"	"	"	"	1%	4%	+E10 1,570	+E7 538	+E5 18.5	+E3 9.71	6.86	5.32

10.1	.5	.5	NA	.5	1%	2%	3,970 +E2	1,270	20.1	9.45	6.75	5.27
10.2	"	"	"	"	2%	4%	2,310 +E26	5,470 +E23	1,140 +E21	2,090 +E18	3,360 +E15	4,680 +E12
10.3	"	"	"	"	1%	4%	2,230 +E3	6,560	36.6	9.92	6.91	5.34
11.1	.5	.48	NA	NA	1%	2%	1,710 +E2	607	18.1	9.43	6.74	5.27
11.2	"	"	"	"	2%	4%	5,510 +E17	1,330 +E15	2,830 +E11	5,310 +E9	8,720 +E6	1,250 +E4
11.3	"	"	"	"	1%	4%	7,460 +E2	2,370	25.2	9.87	6.90	5.34
12.1	.8	.5	NA	NA	1%	2%	3,090 +E4	8,390 +E1	227	10.1	6.78	5.28
12.2	"	"	"	"	2%	4%	2,000 +E35	4,740 +E32	9,900 +E29	1,820 +E27	2,920 +E24	4,070 +E21
12.3	"	"	"	"	1%	4%	2,460 +E5	6,560 +E2	1,610	13.6	6.95	5.35
13.1	.5	.5	.5	NA	1%	2%	2,710	52.8	15.3	9.28	6.70	5.25
13.2	"	"	"	"	2%	4%	1,230 +E3	3,680	27.0	9.38	6.65	5.19
13.3	"	"	"	"	1%	4%	5,980	76.1	16.7	9.66	6.85	5.32

has the highest present value. The present values for the L alternatives in the various models, with various parameters, are therefore set forth at a range of discount factors that span all conceivable choices.

Unlike the mathematician, however, the economist may not simply present the entire range of logical possibilities and leave it at that. For one thing, the logical range encompasses an infinity of discrete possibilities--negative discount rates, and positive rates to infinity; unbounded ranges of parameter values; a large or infinite variety of model specifications; and so on. Instead, the economist must judge which are the economically meaningful alternatives, and consider the implications of them in their constrained variety. The economist ought also to see whether there are general conclusions that may be drawn from the meaningful set of alternatives as a whole.

Few economists or policy makers would agree with Frank Ramsey that any discount rate above zero simply shows a want of imagination. On the other hand, few or no economists would suggest that the public discount rate should be higher than the real private discount rate for projects with the same risk. These considerations should provide agreed-upon boundaries for the appropriate discount rate.

These are some specific findings:

1. In all sets of runs, higher population growth has a higher present value at a zero or low rate of discount, and up to quite substantial discount rates--5% at a minimum--even with parameters that are unreasonably unfavorable to this outcome. This finding may be contrasted with the conclusion of main-stream growth theory that lower population growth is better across the board, and even that negative population growth

is better than population stationarity. This suggests that--putting aside both the other positive and negative effects of population growth such as transfer payments to retirees, environmental adjustment costs, public child services and the like--higher population growth is a good thing in the MDC world, given the value judgments about the discount rate that are implicit in our other social decisions.

2. In all sets of runs with functions that are theoretically reasonable, higher population growth rates have lower present values at some rate of discount--starting somewhere between 5% and 10%. But no matter how high the discount rate, lower population growth's advantage over higher population growth is somewhere between negligible and non-existent. This is because the consumption advantage due to less capital dilution that lower population growth yields in the early years is a very slight advantage at best. This implies that there is no meaningful risk argument against higher population growth, in the context of this model.

3. Together, the advantage of higher population growth at low discount rates, and its non-disadvantage at high discount rates, suggests that a strategy of higher population growth dominates a strategy of lower population growth.

#### SUMMARY AND CONCLUSIONS

Population policy discussions have been influenced by conventional steady-state growth theory in which technical progress is exogenous, implying that faster population growth causes a lower rate of consumption. Making technical progress endogenous in the manner of Verdoorn, Kaldor, Arrow, or Phelps, however, is shown to lead to the opposite implication,

that higher population growth causes a higher rate of growth of consumption, though this inference has not been drawn by the authors of these models.

Steady-state equilibrium analysis is not appropriate for policy decisions, though, because when a nation chooses one or another population growth rate, it begins with the same endowment of capital and people and techniques no matter what the population growth rate chosen. Therefore the appropriate analysis is one which compares the result of two or more growth rates beginning from that initial position. And the logical decision criterion is a present-value comparison of per-capita consumption streams.

The paper analyzes the supply and demand of knowledge, and on those considerations derives the most plausible technical progress functions, which turn out to be mostly variations on a function of Phelps. The effects of various rates of population growth are then simulated with these variations specification and parameters.

The chief result is that with virtually every variant, faster population growth shows better consumption results with discount rates up to 5-10%, a level which is far above the long-run adjusted riskless rate; at higher discount rates lower (or negative) population growth rates have higher present values. If pensions were brought into the analysis, a la Modigliani, Tobin, and Samuelson, higher population growth rates would seem even more beneficial at low discount rates, and would have an advantage over lower population growth rates at discount rates even higher than 6-10%. And even at very high discount rates, lower population growth rates imply present values only a bit above those for higher

population growth rates. The advantage is overwhelmingly with higher population growth in this growth-theoretic analysis.

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## Faculty Working Papers

ANOTHER LOOK AT INDUSTRY GROWTH PATTERNS

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Notes

<sup>1</sup>We required the firms to be listed during the entire sample period. The Center for Security Price Research (CRSP) monthly tape was used to select NYSE listed firms. A firm was considered listed if it had monthly stock returns available for the entire sample period.

<sup>2</sup>The absolute percentage error is computed as the average of  $\left| \frac{\text{Actual EPS} - \text{Predicted EPS}}{\text{Actual EPS}} \right|$ . Since this error metric can be explosive when the denominator approaches zero we truncated errors in excess of ten to a value of ten. This operation was done for a very small percentage of the cases.









